Feuille 3.

[2] 1) $(\frac{1}{2},0)$:

$$(N_+)_i = \frac{J_i + iK_i}{2} = \frac{\sigma_i}{2} \qquad J_i = \frac{\sigma_i}{2}$$

$$(N_-)_i = \frac{J_i - iK_i}{2} = 0 \qquad K_i = -i\frac{\sigma_i}{2}$$

$(0,\frac{1}{2})$

$$(N_+)_i = 0 \qquad J_i = \frac{\sigma_i}{2}$$

$$(N_-)_i = \frac{\sigma_i}{2} \qquad K_i = i\frac{\sigma_i}{2}$$

2) $M_L(\theta,\varphi) = e^{i\frac{\sigma_i}{2}(\theta_i - i\varphi_i)} = e^{\frac{\sigma_i}{2}(\varphi_i + i\theta_i)}$

$M_R(\theta,\varphi) = e^{i\frac{\sigma_i}{2}(\theta_i + i\varphi_i)} = e^{\frac{\sigma_i}{2}(-\varphi_i + i\theta_i)}$

3) Les deux représentations sont équivalentes si il existe U (inversible) telle que $M_R(\theta,\varphi) = U R_L(\theta,\varphi) U^{-1}$ pour tout θ,φ.

Transfo infinitésimale :

$$\mathbb{1} + i\frac{\sigma_i}{2}(\theta_i + i\varphi_i) = U\left(\mathbb{1} + i\frac{\sigma_i}{2}(\theta_i - i\varphi_i)\right)U^{-1}$$

Doit être vrai $\forall\, \theta_i, \varphi_i$ (infinitésimaux) $\Rightarrow$

en utilisant la partie en θ_i : $\sigma_i = U\sigma_i U^{-1}$

φ_i : $\sigma_i = -U\sigma_i U^{-1}$

$\Big\}$ Aucune matrice U ne satisfait ces 2 relations

M_R et M_L ne sont pas équivalentes

4] $M_L^{-1}(\theta,\varphi) = M_L(-\theta,-\varphi)$

$$M_L^+(\theta,\varphi) = e^{\frac{\sigma_i^+}{2}(\varphi_i - i\theta_i)} = M_L(-\theta,\varphi) \qquad (\sigma_i^+ = \sigma_i$$

$$(M_L^+)^{-1}(\theta,\varphi) = M_L(\theta,-\varphi) = M_R(\theta,\varphi)$$

5] $\psi_L'^+ \cdot \chi_R' = (M_L\psi_L)^+ \; M_R\chi_R$

$$= \psi_L^+ \; M_L^+ M_R \; \chi_R$$

$$= \psi_L^+ \; M_L^+ (M_L^+)^{-1} \chi_R = \psi_L^+ \cdot \chi_R$$

invariant ! $\Rightarrow$ scalaire.

6] $\psi_L'^* = (M_L^*(\theta,\varphi)\,\psi_L^*)$

$$= e^{\frac{\sigma_i^*}{2}(\varphi_i - i\theta_i)} \psi_L^*$$

$\sigma_2\sigma_1\sigma_2 = i\sigma_2\sigma_3 = -\sigma_1 = -\sigma_1^*$ OK.

$\sigma_2\sigma_2\sigma_2 = \sigma_2 = -\sigma_2^*$ OK.

$\sigma_2\sigma_3\sigma_2 = i\sigma_1\sigma_2 = -\sigma_3 = -\sigma_3^*$ OK.

$$(\sigma_2\psi_L^*)' = \sigma_2 \, e^{\frac{\sigma_i^*}{2}(\varphi_i - i\theta_i)} \psi_L^*.$$

$$= \sigma_2 \sum \frac{1}{n!}\left(\frac{\sigma_i^*}{2}\right)^n (\varphi_i - i\theta_i)^n \sigma_2 \, \sigma_2 \psi_L^*$$

$$\sigma_2 \sigma_i^{*n} \sigma_2 = \left(\sigma_2 \sigma_i^* \sigma_2\right)^n = (-\sigma_i)^n.$$

$$\left(\sigma_2 \psi_L^*\right)' = e^{-\frac{\sigma_i}{2}(\varphi_i - i\theta_i)} \sigma_2 \psi_L^*$$

$$= e^{\frac{\sigma_i}{2}(i\theta_i - \varphi_i)} \sigma_2 \psi_L^*$$

$$= M_R \left(\sigma_2 \psi_L^*\right)$$

Se transforme comme spinor droit!

$$\left(\sigma_2 \chi_R^*\right)' = \sigma_2 M_R^*(\theta,\varphi) \chi_R^*$$

$$= \sigma_2 e^{\frac{\sigma_i^*}{2}(-\varphi_i - i\theta_i)} \sigma_2 \sigma_2 \chi_R^*$$

$$= e^{-\frac{\sigma_i}{2}(-\varphi_i - i\theta_i)} \sigma_2 \chi_R^*$$

$$= M_L(\theta,\varphi) \left(\sigma_2 \chi_R^*\right)$$

Se transforme comme spinor gauche.

$\boxed{3}$ 1]

$$\psi_L' = M_L(\theta,\varphi)\psi_L.$$

$$\left(\psi_L^+\right)' = \psi_L^+ M_L^+(\theta,\varphi).$$

$$= \psi_L^+ M_R^{-1}(\theta,\varphi).$$

$$= \psi_L^+ M_R(-\theta,-\varphi).$$

2]

$$\left(\psi_L^+ \tilde{\Sigma}^\mu \chi_L\right)' = \psi_L^+ \left(\mathbb{1} + \frac{\sigma_i}{2}(\varphi_i - i\theta_i)\right) \tilde{\Sigma}^\mu \times \left(\mathbb{1} + \frac{\sigma_i}{2}(\varphi_i + i\theta_i)\right) \chi_L$$

$$= \psi_L^+ \tilde{\Sigma}^\mu \chi_L + \frac{i\theta_i}{2} \psi_L^+ \left(\tilde{\Sigma}^\mu \sigma_i - \sigma_i \tilde{\Sigma}^\mu\right) \chi_L + \frac{\varphi_i}{2} \psi_L^+ \left(\tilde{\Sigma}^\mu \sigma_i + \sigma_i \tilde{\Sigma}^\mu\right) \chi_L$$

$$+ O(\theta^2, \varphi^2, \theta\varphi).$$

$$\left(\psi_L^+ \tilde{\Sigma}^0 \chi_L\right)' = \psi_L^+ \tilde{\Sigma}^0 \chi_L + \varphi_i \psi_L^+ \sigma_i \chi_L$$

$$= \psi_L^+ \tilde{\Sigma}^0 \chi_L - \varphi_i \psi_L^+ \tilde{\Sigma}^i \chi_L$$

$$\left(\psi_L^+ \tilde{\Sigma}^i \chi_L\right)' = \psi_L^+ \tilde{\Sigma}^i \chi_L - \frac{i\theta_j}{2} \psi_L^+ (\sigma_i \sigma_j - \sigma_j \sigma_i) \chi_L$$

$$- \frac{\varphi_j}{2} \psi_L^+ (\sigma_i \sigma_j + \sigma_j \sigma_i) \chi_L$$

$$= \psi_L^+ \tilde{\Sigma}^i \chi_L + \varepsilon_{ijk} \theta_j \psi_L^+ \sigma_k \chi_L - \varphi_i \psi_L^+ \tilde{\Sigma}^0 \chi_L.$$

$$= \psi_L^+ \tilde{\Sigma}^i \chi_L - \varepsilon_{ijk} \theta_j \psi_L^+ \tilde{\Sigma}^k \chi_L - \varphi_i \chi_L^+ \tilde{\Sigma}^0 \chi_L$$

même relation de transformation que pour un 4-vecteur $\Rightarrow$

$$= \quad '' \quad + \varepsilon_{ijk} \theta_k \psi_L^+ \tilde{\Sigma}^k \chi_L \quad - \quad ''$$

$\psi_L^+ \tilde{\Sigma}^\mu \chi_L$ se transforme comme un 4-vecteur !

$$\left(\psi_R^+ \Sigma^\mu \chi_R\right)' = \psi_R^+ \left(M_R^+ \Sigma^\mu M_R\right) \chi_R.$$

$$= \psi_R^+ \left(M_L(-\theta, -\varphi) \Sigma^\mu M_R(\theta, \varphi) \right) \chi_R$$

$$= \psi_R^+ \left(\left(\mathbb{1} + \frac{\sigma_i}{2}(-\varphi_i - i\theta_i)\right) \Sigma^\mu \left(\mathbb{1} + \frac{\sigma_i}{2}(-\varphi_i + i\theta_i)\right)\right) \chi_R$$

$$= \psi_R^+ \sigma^\mu \chi_R + \frac{i\theta_i}{2} \psi_R^+ \left(\Sigma^\mu \sigma_i - \sigma_i \Sigma^\mu\right) \chi_R$$

$$- \frac{\varphi_i}{2} \psi_R^+ (\sigma_i \Sigma^\mu + \Sigma^\mu \sigma_i) \chi_R$$

$$\left(\psi_R^\dagger \Sigma^0 \chi_R\right)' = \psi_R^\dagger \Sigma^0 \chi_R - \varphi_i \, \psi_R^\dagger \Sigma^i \chi_R.$$

$$\left(\psi_R^\dagger \Sigma^i \chi_R\right)' = \psi_R^\dagger \Sigma^i \chi_R + \frac{i\theta_j}{2} \psi_R^\dagger \left(\sigma_i \sigma_j - \sigma_j \sigma_i\right) \chi_R.$$
$$- \frac{\varphi_j}{2} \psi_R^\dagger \left(\sigma_j \sigma_i + \sigma_i \sigma_j\right) \chi_R.$$

$$= \psi_R^\dagger \Sigma^i \chi_R - \varepsilon_{ijk} \theta_j \psi_R^\dagger \Sigma^k \chi_R - \varphi_i \psi_R^\dagger \Sigma^0 \chi_R$$
$$= \quad \text{''} \quad + \varepsilon_{ijk} \theta_k \psi_R^\dagger \Sigma^j \chi_R - \quad \text{''}$$

$\psi_R^\dagger \Sigma^\mu \chi_R$ se transforme comme un 4-vecteur.

<u>Remarques</u>

* On introduit Σ^μ au lieu de la notation standard σ^μ.

car si non $\sigma^i = \sigma_i = -\sigma_i$ ⇐ Ambiguë
$\sigma^{(\mu=i)} \qquad = -\sigma_{(\mu=i)}$.

alors que $\Sigma^i = \sigma_i = -\Sigma_i$ ⇐ pas Ambiguë

* Pourquoi prend-on $\psi_R^\dagger$ et χ_R si on veut travailler avec $(\frac{1}{2}, \frac{1}{2})$

parce que $\psi_R^\dagger = \left(\psi_R^*\right)^t \simeq \psi_L^t$

Quand on travaille avec $\psi_R^\dagger$ et χ_R, on fait des combinaisons de ψ_L et de χ_R.

$\boxed{4}$ 1] dimensión 3.

2] $(1,0)$: $(N_+)_i = j_i = \frac{1}{2}(J_i + i K_i)$

$(N_-)_i = 0 = \frac{1}{2}(J_i - i K_i)$.

$J_i = j_i$

$K_i = -i j_i$

$$M = e^{\vec{j}(i\vec{\theta} + \vec{\varphi})} = e^{j_i(i\theta_i + \varphi_i)}$$

$$= \mathbb{1} + (i\theta_i + \varphi_i)\, j_i + O(\theta^2, \varphi^2, \theta_i\varphi).$$

3] $R'_i + iC'_i = \left(\delta_{ij} + (i\theta_k + \varphi_k)(-i\,\varepsilon_{ijk})\right)\left(R_j + iC_j\right)$.

$$= \left\{R_i + \varepsilon_{ijk}\theta_k R_j + \varepsilon_{ijk}\varphi_k C_j\right\}$$
$$+ i\left\{C_i - \varepsilon_{ijk}\varphi_k R_j + \varepsilon_{ijk}\theta_k C_j\right\}$$

$$R'_i = R_i + \varepsilon_{ijk}\theta_k R_j + \varepsilon_{ijk}\varphi_k C_j$$
$$C'_i = C_i - \varepsilon_{ijk}\varphi_k R_j + \varepsilon_{ijk}\theta_k C_j$$

4] $E'_1 = F'^{10} = \Lambda^1{}_\mu \Lambda^0{}_\nu F^{\mu\nu}$

$$= (g^1{}_\mu + \omega^1{}_\mu)(g^0{}_\nu + \omega^0{}_\nu)F^{\mu\nu}$$

$$= F^{10} + \omega^1{}_\mu F^{\mu 0} + \omega^0{}_\nu F^{1\nu}$$

$$= E_1 \qquad + \theta_3 E_2 - \theta_2 E_3 + \varphi_2 B_3 - \varphi_3 B_2 .$$

$$\Rightarrow \quad E'_i = E_i + \varepsilon_{ijk}\theta_k E_j - \varepsilon_{ijk}\varphi_k B_j$$

$$B'_1 = F'^{32} = \Lambda^3{}_\mu \Lambda^2{}_\nu F^{\mu\nu}$$

$$= (g^3{}_\mu + \omega^3{}_\mu)(g^2{}_\nu + \omega^2{}_\nu) F^{\mu\nu}$$

$$= F^{32} + \omega^3{}_\mu F^{\mu 2} + \omega^2{}_\nu F^{3\nu}$$

$$= B_1 + \phi_3 E_2 - \theta_2 B_3 - \phi_2 E_3 + \theta_3 B_2 .$$

$$\Rightarrow \quad B'_i = B_i + \varepsilon_{ijk}\theta_k B_j + \varepsilon_{ijk}\varphi_k E_j$$

5] B se transforme comme R ou E $-R$

E "   " C B C

↳ $\vec{R} + i\vec{C}$ se transforme comme un tenseur antisymétrique. $F^{\mu\nu}$.

6] $(0,1)$

$$M = e^{j_i(i\theta_i - \varphi_i)} = \mathbb{1} + j_i(i\theta_i - \varphi_i) + o(\theta^2, \varphi^2, \theta\varphi).$$

Revient à changer φ en $-\varphi$.

$$↳ \quad R'_i = R_i + \varepsilon_{ijk}\theta_k R_j - \varepsilon_{ijk}\varphi_k C_j$$

$$C'_i = C_i + \varepsilon_{ijk}\varphi_k R_j + \varepsilon_{ijk}\theta_k C_j$$

↳ B se transforme comme C ou B se transforme comme $-R$.

E "   " R E "   " C.

$\boxed{5}$ 1] $P\,\Lambda(\theta,\varphi)P = P\,(\mathbb{1} + \omega)\,P$

$$= P\,\mathbb{1}\,P + P\,\omega\,P$$

$$= \mathbb{1} + \begin{pmatrix} 0 & \phi_1 & \phi_2 & \phi_3 \\ \phi_1 & 0 & \theta_3 & -\theta_2 \\ \phi_2 & -\theta_3 & 0 & \theta_1 \\ \phi_3 & \theta_2 & -\theta_1 & 0 \end{pmatrix}$$

$$= \Lambda(\theta,-\varphi).$$

2] Rq: $P\,\omega^n_{(\theta,\varphi)}P = (P\,\omega\,P)^n = \omega(\theta,-\varphi)^n.$

↳ $P\Lambda(\theta,\varphi)P = \Lambda(\theta,-\varphi)$. pour transfo finie.

3] Si M_R est associé à Λ

M_L est associé à $P\Lambda P$. car $\underset{P\Lambda P}{M_L(\theta,-\varphi)} = \underset{\Lambda}{M_R(\theta,\varphi)}$.

4] $(\psi_R)' = (i\sigma_2\psi_L^*)' = i\sigma_2\chi_R^* = -\chi_L$

$\chi_L' = -\psi_R$

5] scalaires sous $\mathcal{L}_+^\uparrow$: $\psi_L^+\chi_R$ — Sous parité: → $\chi_R^+\psi_L$

$\chi_L^+\psi_R$ → $\psi_R^+\chi_L$.

Scalaires : $\psi_L^+\chi_R + \chi_R^+\psi_L$

$\chi_L^+\psi_R + \psi_R^+\chi_L$.

Pseudo-scalaires $\psi_L^+\chi_R - \chi_R^+\psi_L$

$\chi_L^+\psi_R - \psi_R^+\chi_L$

4] $\psi_L^+ \chi_R \xrightarrow{\text{sous Parité}} \psi_R^+ \chi_L$

↳ $\psi_L^+ \chi_R + \psi_R^+ \chi_L$: scalaire

$\psi_L^+ \chi_R - \psi_R^+ \chi_L$: pseudo-scalaire

5] $\psi_L^+ \tilde{\Sigma}^\mu \chi_L \xrightarrow{\text{parité}} \psi_R^+ \tilde{\Sigma}^\mu \chi_R$.

$$= P^\mu{}_\nu \, \psi_R^+ \Sigma^\nu \chi_R$$

$\psi_R^+ \Sigma^\mu \chi_R \longrightarrow \psi_L^+ \Sigma^\mu \chi_L$.

$$= P^\mu{}_\nu \, \psi_L^+ \tilde{\Sigma}^\nu \chi_L.$$

$A^\mu = \psi_L^+ \tilde{\Sigma}^\mu \chi_L$

$B^\mu = \psi_R^+ \Sigma^\mu \chi_R$

$(A^\mu + B^\mu) \longrightarrow (P^\mu{}_\nu B^\nu + P^\mu{}_\nu A^\nu) = P^\mu{}_\nu (A^\nu + B^\nu)$. ← 4-vecteur.

$(A^\mu - B^\mu) \longrightarrow -P^\mu{}_\nu (A^\nu - B^\nu)$ ← pseudo-4vecteur.

$\boxed{6}$

1) $\frac{1}{2} \mathrm{Tr}\, \Sigma^0 \tilde{\Sigma}^0 = 1 = g^{00}$

$\frac{1}{2} \mathrm{Tr}\, \Sigma^0 \tilde{\Sigma}^i = -\frac{1}{2} \mathrm{Tr}\, \sigma_i = 0 = g^{0i}$

$\frac{1}{2} \mathrm{Tr}\, \Sigma^i \tilde{\Sigma}^0 = 0 = g^{i0}$

$\frac{1}{2} \mathrm{Tr}\, \Sigma^i \tilde{\Sigma}^j = -\frac{1}{2} \mathrm{Tr}\, \sigma_i \sigma_j = -\delta_{ij} = g^{ij}$

2) $\begin{pmatrix} a & c+id \\ c-id & b \end{pmatrix} \Rightarrow$ 4 réels.

$$X = \begin{pmatrix} x^0+x^3 & x^1-ix^2 \\ x^1+ix^2 & x^0-x^3 \end{pmatrix}$$

3) $\det X = (x^0)^2 - (x^3)^2 - (x^1)^2 - (x^2)^2 = g_{\mu\nu} x^\mu x^\nu$

4) Matrices 2×2 : 8 paramètres réels.
$\det = 1 \Rightarrow$ 2 contraintes $\Rightarrow$ 6 paramètres.

5) $(M X M^+)^+ = M X^+ M^+ = M X M^+$

$\hookrightarrow$ $M X M^+ = x'^\mu \Sigma_\mu$

a $x'^\mu x'_\mu = \det M X M^+ = \det M \det X \det M^+$
$= \det X.$

$= x^\mu x_\mu.$

$\hookrightarrow$ x'^μ obtenu par transfo de Lorentz à partir de x^μ.

$x'^\mu = \Lambda(M)^\mu{}_\nu x^\nu.$

6] $$N M X M^+ N^+ = (NM) X (NM)^+ = x''^\mu \Sigma_\mu$$

avec $x''^\mu = \Lambda(NM)^\mu{}_\nu x^\nu$.

$$N M X M^+ N^+ = N (M X M^+) N^+$$
$$= N \; x'^\mu \Sigma_\mu N^+$$

$$x''^\mu = \Lambda(N)^\mu{}_\nu x'^\nu = \Lambda(N)^\mu{}_\rho \Lambda^\rho{}_\nu(M) x^\nu$$

$\hookrightarrow$ $\Lambda(NM) = \Lambda(N)\Lambda(M)$. homomorphisme.

7] $$x'^\mu \Sigma_\mu = M X M^+ = M x^\rho \Sigma_\rho M^+.$$

$$\frac{1}{2} \mathrm{Tr}\left(x'^\mu \Sigma_\mu \tilde{\Sigma}^\nu\right) = \frac{1}{2} \mathrm{Tr}\left(M x^\rho \Sigma_\rho M^+ \tilde{\Sigma}^\nu\right).$$

$$x'^\nu = \frac{1}{2} \mathrm{Tr}\left(M \Sigma_\rho M^+ \tilde{\Sigma}^\nu\right) x^\rho.$$

$$\Lambda(M)^\mu{}_\nu = \frac{1}{2} \mathrm{Tr}\left(M \Sigma_\nu M^+ \tilde{\Sigma}^\mu\right)$$

8] $$\Lambda^\mu{}_\nu = \frac{1}{2} \mathrm{Tr}\left[\left(\mathbb{1} + \frac{\sigma_k}{2}(i\theta_k + \varphi_k)\right)\Sigma_\nu \left(\mathbb{1} + \frac{\sigma_k}{2}(-i\theta_k + \varphi_k)\right)\tilde{\Sigma}^\mu\right]$$
$$= g^\mu{}_\nu + \frac{i\theta_k + \varphi_k}{4} \mathrm{Tr}\left(\sigma_k \Sigma_\nu \tilde{\Sigma}^\mu\right).$$
$$+ \frac{-i\theta_k + \varphi_k}{4} \mathrm{Tr}\left(\Sigma_\nu \sigma_k \tilde{\Sigma}^\mu\right).$$

$$\Lambda^0{}_i = \frac{i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left(\sigma_k(-\sigma_i)\right) + \frac{-i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left((-\sigma_i)\sigma_k\right)$$

$$= -\varphi_k \qquad \text{OK}$$

$$\Lambda^i{}_0 = \frac{i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left(\sigma_k(-\sigma_i)\right) + \frac{-i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left((-\sigma_i)\sigma_k\right)$$

$$= -\varphi_k \qquad \text{OK}$$

$$\Lambda^i{}_j = \frac{i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left(\sigma_k(-\sigma_j)(-\sigma_i)\right) + \frac{-i\sigma_k+\varphi_k}{4}\,\mathrm{Tr}\left((-\sigma_j)\sigma_k(-\sigma_i)\right)$$

$$= (\qquad)\,\mathrm{Tr}\left(i\varepsilon_{jkl}\sigma_l(-\sigma_i)\right) + (\qquad)\,\mathrm{Tr}\left(i\varepsilon_{jkl}\sigma_l(\sigma_i)\right)$$

$$= -2i\,\varepsilon_{jki}\,\frac{i\sigma_k+\varphi_k}{4} + \frac{-i\sigma_k+\varphi_k}{4}\,2i\,\varepsilon_{jki}$$

$$= \varepsilon_{ijk}\,\sigma_k \qquad \text{OK}.$$

g) Maintenant : $X = x^\mu \tilde{\Sigma}_\mu$

$$\hookrightarrow \Lambda_R(M)^\mu{}_\nu = \frac{1}{2}\,\mathrm{Tr}\left(M\tilde{\Sigma}_\nu M^+ \Sigma^\mu\right)$$

$$= \frac{1}{2}\,\mathrm{Tr}\left(M^+\Sigma^\mu\tilde{\Sigma}_\nu M\right) = \Lambda(M^+)^\mu{}_\nu$$

$$= \Lambda_L^{-1}(M^+)^\mu{}_\nu$$

$$= \Lambda_L\left((M^+)^{-1}\right)^\mu{}_\nu$$

$$\text{or } \left(M_R^+(\sigma,\varphi)\right)^{-1} = M_L(\sigma,\varphi) \text{ compatible avec } \Lambda_L.$$

$\hookrightarrow$ Λ_R compatible avec M_R.

7

1) Avec le bon signe :

$$R_x(\tfrac{\pi}{2}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_y(\tfrac{\pi}{2}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}.$$

$$R_x(\tfrac{\pi}{2}) R_y(\tfrac{\pi}{2}) R_x(-\tfrac{\pi}{2}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_z(\tfrac{\pi}{2}).$$

$[-J_i, -J_j] = -i\varepsilon_{ijk}(-J_k)$

↳ pas les bons facteurs de structure.

$$T_x(\tfrac{\pi}{2}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}.$$

$$T_y(\tfrac{\pi}{2}) = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

$$T_x(\tfrac{\pi}{2}) T_y(\tfrac{\pi}{2}) T_x(-\tfrac{\pi}{2}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = T_z(-\tfrac{\pi}{2}).$$

↳ ne respecte pas la loi de groupe ⇒ pas un homomorphisme.

2) $\sigma_1^* = \sigma_1 \quad \sigma_2^* = -\sigma_2 \quad \sigma_3^* = \sigma_3$

$\tau_1 = -\sigma_1 \quad \tau_2 = \sigma_2 \quad \tau_3 =$

$$\tau_1^2 = \tau_2^2 = \tau_3^2 = \mathbb{1}.$$

$$\tau_1 \tau_2 = -\sigma_1\sigma_2 = -i\sigma_3 = i\tau_3$$

$$\tau_2 \tau_3 = -\sigma_2\sigma_3 = -i\sigma_1 = i\tau_1$$

$$\tau_3 \tau_1 = \sigma_3\sigma_1 = i\sigma_2 = i\tau_2.$$

↳ $\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i\varepsilon_{ijk}\frac{\tau_k}{2}.$

↳ $e^{i\frac{\theta}{2}\vec{n}\vec{\tau}}$ est une représentation

3] Peut-on trouver U /

$$U\, e^{i\frac{\theta}{2}\hat{n}\vec{\tau}}\, U^{-1} = e^{i\frac{\theta}{n}\hat{n}\vec{\sigma}} \qquad \forall \theta, \hat{n}\ ?$$

pour θ infinitésimal, on doit avoir

$$U\tau_i U^{-1} = \sigma_i$$
$$U\sigma_i^* U^{-1} = -\sigma_i$$
$$\hookrightarrow \sigma_i^* = -U^{-1}\sigma_i\, U$$

$U = \sigma_2$ fait le taf. (voir $\boxed{2}$).

et ça marche pour θ fini ?

$$\sigma_2\, e^{i\frac{\theta}{2}\hat{n}\vec{\tau}}\, \sigma_2 = \sum_n \frac{1}{n!}\, \sigma_2 \left(i\frac{\theta}{2}\,\hat{n}\vec{\tau}\right)^n \sigma_2.$$
$$= \sum_n \frac{1}{n!} \left(i\frac{\theta}{2}\,\hat{n}\ \sigma_2\vec{\tau}\sigma_2\right)$$
$$= e^{i\frac{\theta}{2}\hat{n}\vec{\sigma}}$$

$\Rightarrow$ OUI, ça marche pour θ fini...