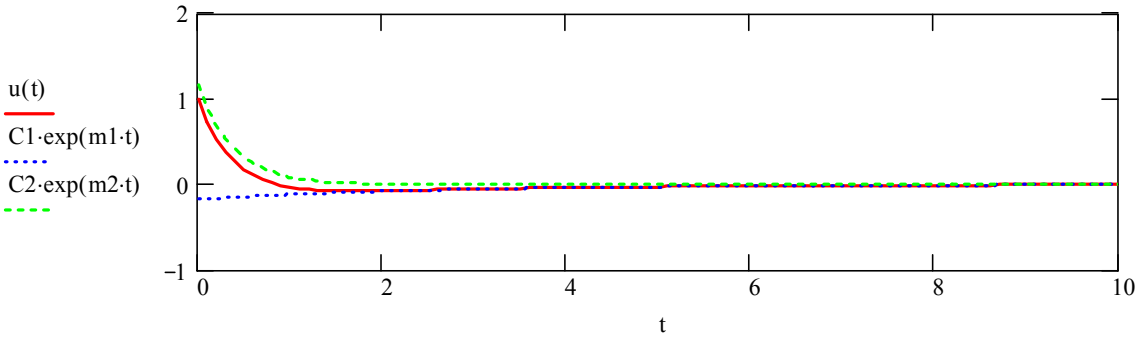
1. Régime transitoire de l'oscillateur amorti : solution de l'équation homogène

a) L'équation caractéristique admet deux solutions réelles.

Conditions initiales: x(0)=x0 et x'(0)=0 $\quad C1 := \frac{m1}{m1 - m2} \cdot x0 \quad C2 := \frac{-m2}{m1 - m2} \cdot x0$

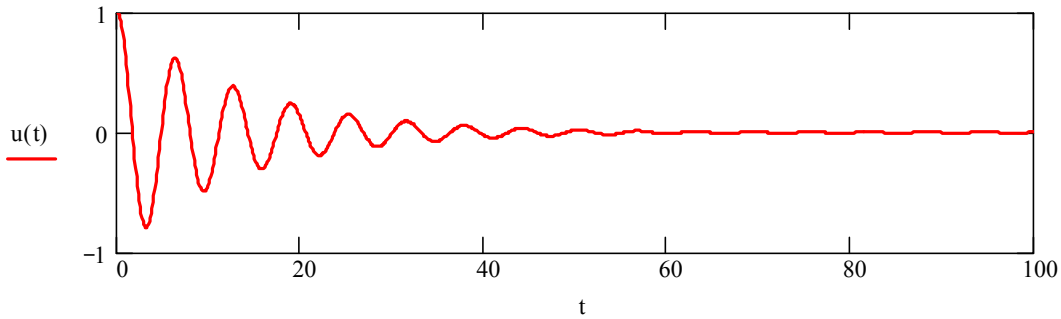
$u(t) := C1 \cdot \exp(m1 \cdot t) + C2 \cdot \exp(m2 \cdot t) \qquad u(0) = 1 \qquad t := 0, 0.1 .. 10$



b) L'équation caractéristique admet deux solutions complexes conjuguées.

Conditions initiales: x(0)=x0 et x'(0)=0 $\quad \mu := Re(m1) \quad \sigma := Im(m1) \quad C1 := x0 \quad C2 := x0 \cdot \left(\frac{-\mu}{\sigma}\right)$

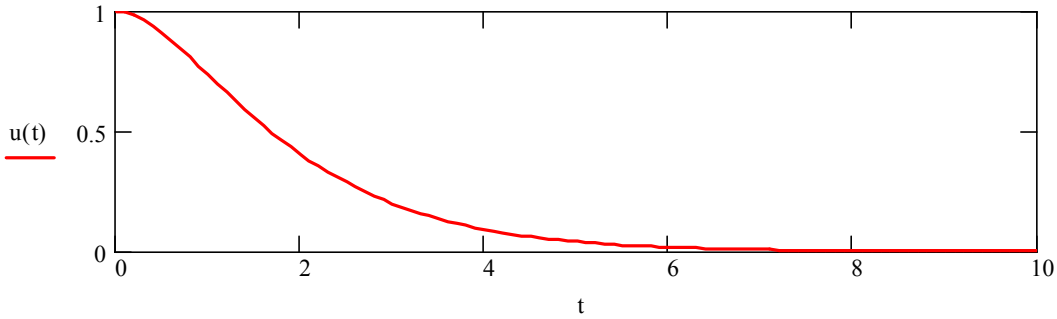
$u(t) := \exp(\mu \cdot t) \cdot (C1 \cdot \cos(\sigma \cdot t) + C2 \cdot \sin(\sigma \cdot t)) \qquad u(0) = 1 \qquad t := 0, 0.1 .. 100$



c) L'équation caractéristique admet une solution réelle double

Conditions initiales: x(0)=x0 et x'(0)=0 $\quad C1 := x0 \quad C2 := -m1 \cdot x0$

$u(t) := \exp(m1 \cdot t) \cdot (C1 + C2 \cdot t) \qquad u(0) = 1 \qquad t := 0, 0.1 .. 10$



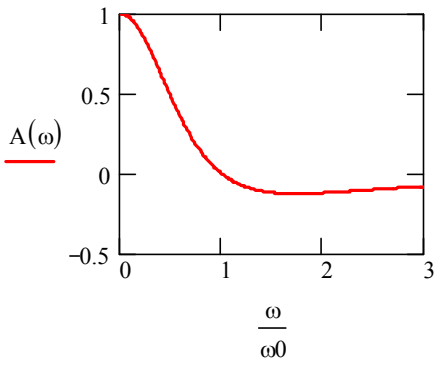
2. Régime permanent de l'oscillateur amorti : solution particulière en régime harmonique

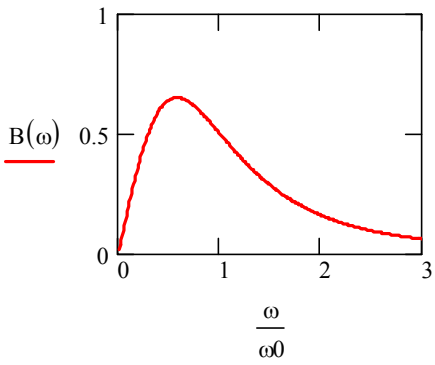
Equation : $x'' + \gamma x' + \omega_0^2 x = (F0/m)\cos(\omega t)$

Solution particulière de la forme : $y(t) = A\cos(\omega t) + B\sin(\omega t)$

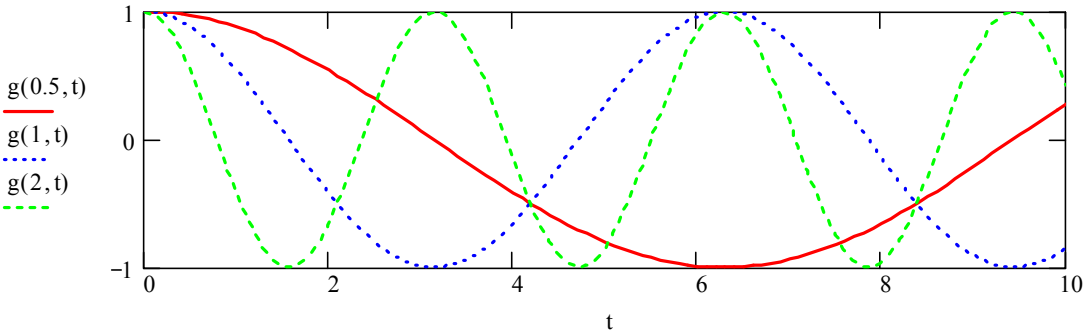
$$A(\omega) := \frac{-F0\cdot\left(\omega^2 - \omega0^2\right)}{m\cdot\left(\omega^4 - 2\cdot\omega^2\cdot\omega0^2 + \gamma^2\cdot\omega^2 + \omega0^4\right)} \qquad B(\omega) := \frac{\gamma\cdot(F0\cdot\omega)}{m\cdot\left(\omega^4 - 2\cdot\omega^2\cdot\omega0^2 + \gamma^2\cdot\omega^2 + \omega0^4\right)}$$

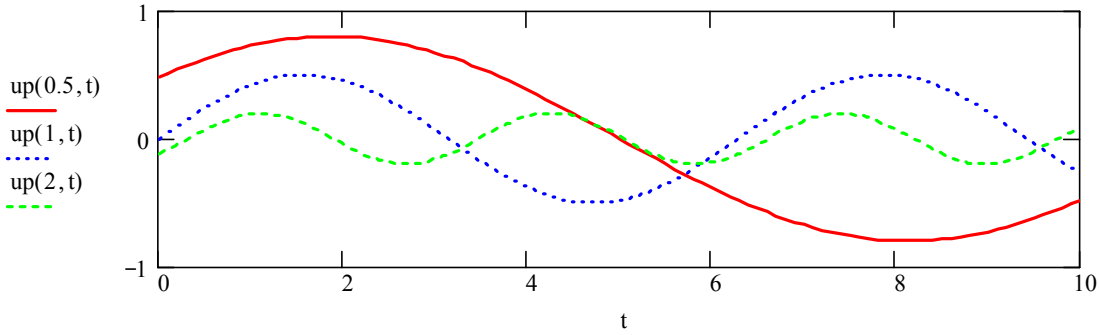
$m = 1 \qquad \gamma = 2 \qquad \omega0 = 1 \qquad F0 = 1 \qquad \omega := 0.01, 0.02 .. 3$





$up(\omega, t) := A(\omega)\cdot\cos(\omega\cdot t) + B(\omega)\cdot\sin(\omega\cdot t)$





3. Superposition des régimes : démarrage de l'oscillateur

Equation : ax" + bx' + cx = 0 $\quad a := 1 \quad b := 0.15 \quad c := 1 \quad m := a \quad \gamma := \frac{b}{m} \quad \omega 0 := \sqrt{\frac{c}{m}}$

Discriminant eq. caractéristique: $\Delta := b^2 - 4 \cdot a \cdot c \qquad \Delta = -3.978$

Solutions: $m1 := \frac{\left(-b + \sqrt{\Delta}\right)}{2 \cdot a} \qquad m2 := \frac{\left(-b - \sqrt{\Delta}\right)}{2 \cdot a} \qquad m1 = -0.075 + 0.997i \qquad m2 = -0.075 - 0.997i$

Conditions initiales: x(0)=x0 et x'(0)=0 $\quad \mu := \mathrm{Re}(m1) \quad \sigma := \mathrm{Im}(m1) \quad C1 := x0 \quad C2 := x0 \cdot \left(\frac{-\mu}{\sigma}\right)$

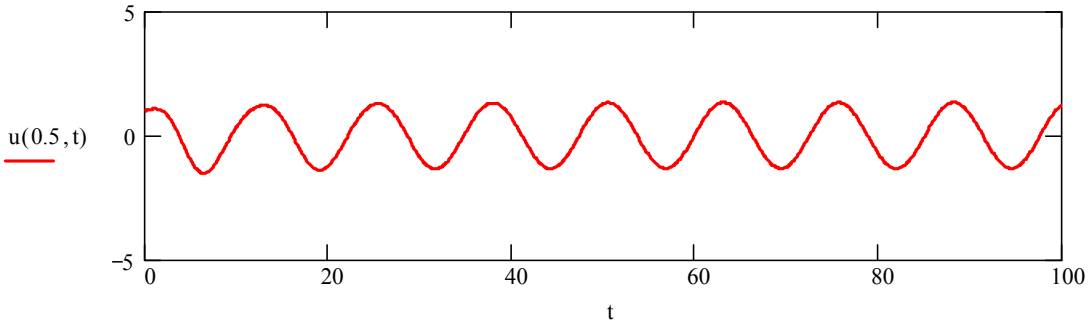
$$A(\omega) := \frac{-F0 \cdot \left(\omega^2 - \omega 0^2\right)}{m \cdot \left(\omega^4 - 2 \cdot \omega^2 \cdot \omega 0^2 + \gamma^2 \cdot \omega^2 + \omega 0^4\right)} \qquad B(\omega) := \frac{\gamma \cdot (F0 \cdot \omega)}{m \cdot \left(\omega^4 - 2 \cdot \omega^2 \cdot \omega 0^2 + \gamma^2 \cdot \omega^2 + \omega 0^4\right)}$$

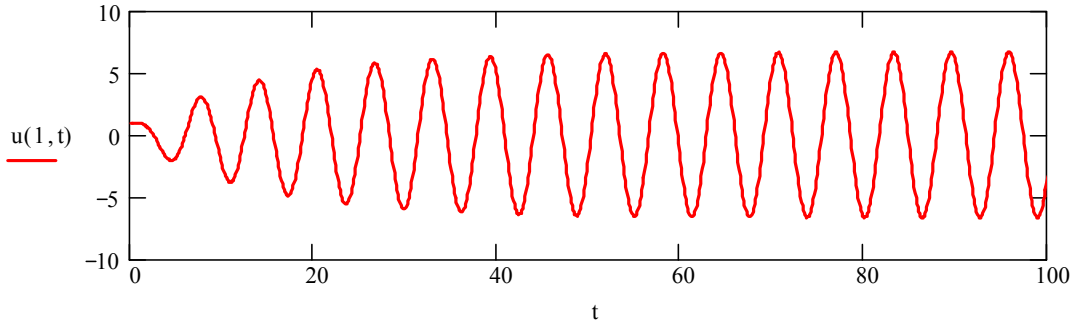
$$up(\omega, t) := A(\omega) \cdot \cos(\omega \cdot t) + B(\omega) \cdot \sin(\omega \cdot t)$$

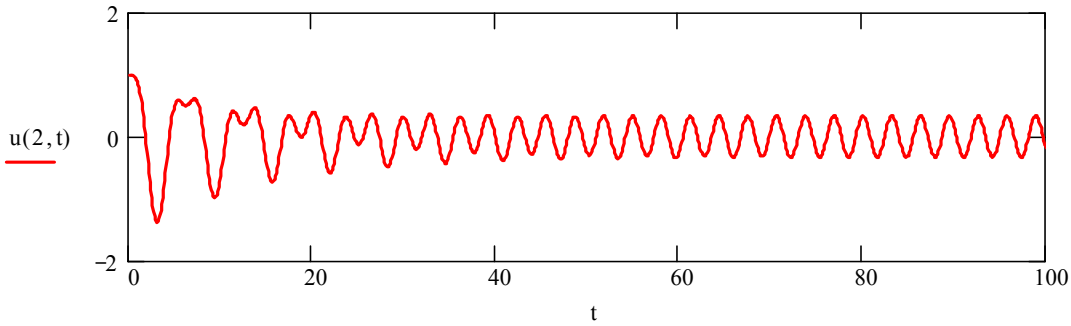
$$x0 := 1 \qquad C1(\omega) := x0 - A(\omega) \qquad C2(\omega) := x0 \cdot \left(\frac{-\mu}{\sigma}\right) - B(\omega) \cdot \frac{\omega}{\sigma}$$

$$uh(\omega, t) := \exp(\mu \cdot t) \cdot (C1(\omega) \cdot \cos(\sigma \cdot t) + C2(\omega) \cdot \sin(\sigma \cdot t))$$

$$u(\omega, t) := up(\omega, t) + uh(\omega, t) \qquad u(1, 0) = 1 \qquad t := 0, 0.1 .. 100$$







4. Régime permanent de l'oscillateur amorti : analyse du spectre de la réponse

$$X(\omega,\gamma) := \frac{F0}{m\cdot\left(\omega0^2 - \omega^2 + j\cdot\gamma\cdot\omega\right)} \qquad F0 = 1 \qquad \omega0 = 1 \qquad m = 1$$